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Vortex growth in jets

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Observations are reported on the growth of vortices in the vortex sheets bounding the jet emerging from a sharp-edged two-dimensional slit and from a sharp-edged circular orifice. A regular periodic flow is observed near the orifice for both configurations when the Reynolds number of the jet lies between about 500 and 3000. The two-dimensional jet produces a symmetric pattern of vortex pairs with a Strouhal number of 0·43. Vortex rings are formed in the circular jet with a Strouhal number of 0·63. Computer experiments show that a growing pair of vortices in two parallel vortex sheets produces a symmetric pattern of vortices upstream from the original disturbance.

Introduction

Symmetric vortex streets, consisting of either two-dimensional vortex pairs or axisymmetric vortex rings, have been studied in some detail both experimentally and analytically during the last forty years, and a fairly extensive literature has now evolved on this subject. There appears to have been two main approaches to the study of this type of vortex street. On the one hand, the occurrence of symmetric vortex streets in the instability phenomena associated with free boundary layers has resulted in their being studied from a strictly fluid mechanics point of view. On the other hand, considerable attention from workers in the field of acoustics has been given to symmetric streets primarily because of their apparent relationship to the operation of certain sound-generating devices, as for example the Rayleigh bird-call. In these devices, the interaction of the vortex-bearing jet with wall boundaries in the flow has a strong effect on the behaviour of the jet and the generation of the vortices.

Among the earliest photographs showing the growth of vortex rings in a circular jet are those of Johansen (1929), who investigated the circular jet produced downstream of a broad orifice plate set in a circular pipe. Brown (1935) published photographs showing the periodic motions of smoke-laden two-dimensional air jets subjected to external acoustic excitation. Most of these photographs show the excited jets to exhibit a sinuous motion accompanied by the development of an alternating vortex pattern, although some of the pictures clearly show symmetric streets. Similar photographs were produced by Andrade (1941) for excited circular jets of water issuing into stationary water.

In the course of experimental studies on hole and slot tones, using various jet-edge geometries, von Gierke (1950) obtained several pictures of symmetric disturbances in smoke-filled air jets emanating from cylindrical orifices. Anderson (1954, 1955*b*, 1956) investigated in depth the relationship between the acoustic frequencies and the vortex growth process of circular jets emanating from relatively thin orifices, presenting photographs of the streets of vortex rings in the jets. He also examined (1955*a*) the flow through a relatively thick cylindrical orifice in which separation of the flow from the upstream edge resulted in symmetric disturbances within the orifice starting just behind the leading edge, causing an oscillating component of flow through the orifice. Again, Chanaud & Powell (1965) reported experiments on hole tones, in which they were primarily interested in the effect of a second orifice on the disturbances in circular jets. They did, however, view a smoke-laden jet with stroboscopic lighting, and observed that the disturbances were unambiguously symmetrical, and that at higher jet velocities the disturbances grew into smoke-ring vortices.

The instability and formation of vortices in the laminar free boundary layers of axisymmetric and two-dimensional jets have been studied extensively by Wille and co-workers at the Institut für Turbulenzforschung in Berlin. Wehrmann & Wille (1958) presented a study of the jets issuing from contoured nozzles, including an illustration of the breakdown of the vortex street in a water jet at a Reynolds number of 10,000. Photographs showing the breakdown of the vortex streets in the free boundary layers emanating from both axisymmetric and plane nozzles are shown in papers by Michalke & Wehrmann (1962) and Michalke & Wille (1964). These photographs are of smoke filaments in air, and include the effects of external excitations on the growth patterns. Pictures taken from a film by Berger showing the vortex street in a jet of water issuing from a pipe into quiescent water are included in Michalke & Wehrmann (1962), Michalke (1964*a*) and Wille (1963). In this last paper are included smoke pictures at high Reynolds number, similar to those in the aforementioned papers by Michalke and co-authors. Freymuth (1966) investigated the acoustically excited free boundary layers of axisymmetric and plane jets using hot-wire techniques. The break-up of a symmetric vortex pattern in the separation layers formed by a plane nozzle when externally disturbed is shown in Michalke & Freymuth (1966), who also compare the rolling up of a smoke-laden free boundary layer with a numerical calculation using the hyperbolic-tangent velocity profile.

Other studies in this field include the work of Sato (1960), who investigated two-dimensional silent and externally excited air jets, identifying both symmetric and antisymmetric velocity fluctuations in the unstable region of the jet. Becker & Massaro (1968) discussed the instability phenomena associated with axisymmetric jets issuing from a contoured nozzle when subjected to minimum background noise, pure tone excitation and intense noise excitation. They include photographs illustrating the vortex growth and break-up processes under the various operating conditions.

The investigations quoted above have been concerned with vortex growth in jets issuing from either nozzles or square-edged thick orifices. The studies involving flow from nozzles have usually been performed at sufficiently high values of the jet Reynolds number to satisfy the condition that the jet boundary-layer thickness is small compared with the jet diameter or width. Under these conditions, it can be shown that the Strouhal number of the shed vortices is proportional to the square root of the jet Reynolds number, and several authors have reported data substantiating this (e.g. Michalke & Wehrmann 1962; Wille 1963; Michalke & Wille 1964; Becker & Massaro 1968). When the Reynolds number is low enough, so that the laminar boundary layer occupies the whole cross-section of the nozzle, the Strouhal number is independent of Reynolds number, as observed by Sato (1960). The behaviour of the vortex system shed from a square-edged orifice is somewhat more difficult to anticipate, since the shedding frequency and wavelength could depend on both the orifice diameter and thickness. Inspection of Anderson's results for jet tones (1954, 1956) indicates that the Strouhal number increases with Reynolds number for orifice diameter-to-thickness ratios of about unity, whereas the Strouhal number appears to be independent of Reynolds number when this ratio is about three.

A third mechanism for the generation of symmetric vortex streets is the sharp-edged slit or circular orifice. As stated by Becker & Massaro (1968), 'the phenomena associated with orifices have been only fragmentarily observed and more work is needed to obtain clarification'. In this paper, we describe flow visualization experiments which have been performed to study the vortex growth and break-up in both two-dimensional and axisymmetric jets emanating from very sharp-edged slits and orifices. The experiments covered a Reynolds number range of approximately 500 to 3000, and were performed in an environment in which all external disturbances were eliminated. The Strouhal numbers for both the plane and axisymmetric jets were found to be independent of the Reynolds number.

The analysis of the stability of laminar free boundary layers has been developed in recent years using inviscid linear hydrodynamic stability theory (e.g. Michalke 1964*b*, 1965; Michalke & Freymuth 1966; Michalke & Timme 1967). Alternatively, attempts to describe the mechanism of the rolling-up of disturbed free shear layers have been made using a numerical approach, in which a free shear layer is replaced by an array of point vortices. In this paper, two computer experiments using this approach are described. These were undertaken in an effort to simulate the vortex growth process observed in the laboratory experiments. Earlier workers have attempted to predict the rolling-up of vortex sheets using numerical methods, but in each case the calculations have been carried out with the restriction that the vortex pattern possessed a predetermined wavelength. No such restriction was imposed on the vortex pattern for the calculations described herein. Two infinite parallel vortex sheets were disturbed, first symmetrically and then non-symmetrically, over a small segment of their length. The paper describes the results of the calculations, in which a growing pattern of symmetric vortices was eventually produced upstream of the original disturbances in both instances.

Apparatus

The experiments were carried out in a water tunnel facility, shown schematically in figure 1. The tunnel consists essentially of two large reservoirs, interconnected by a closed rectangular channel which constitutes the test section. The whole unit is made of plexiglass. Water enters the upstream reservoir from a very large settling chamber through a control valve, which is used to regulate the flow through the whole system. The liquid exits from the downstream reservoir by means of an adjustable overflow weir, whence it is ducted to a two-way valve. One valve exit directs the water to a metering station, while the second exit leads to a dump tank. In normal operation, the liquid is pumped

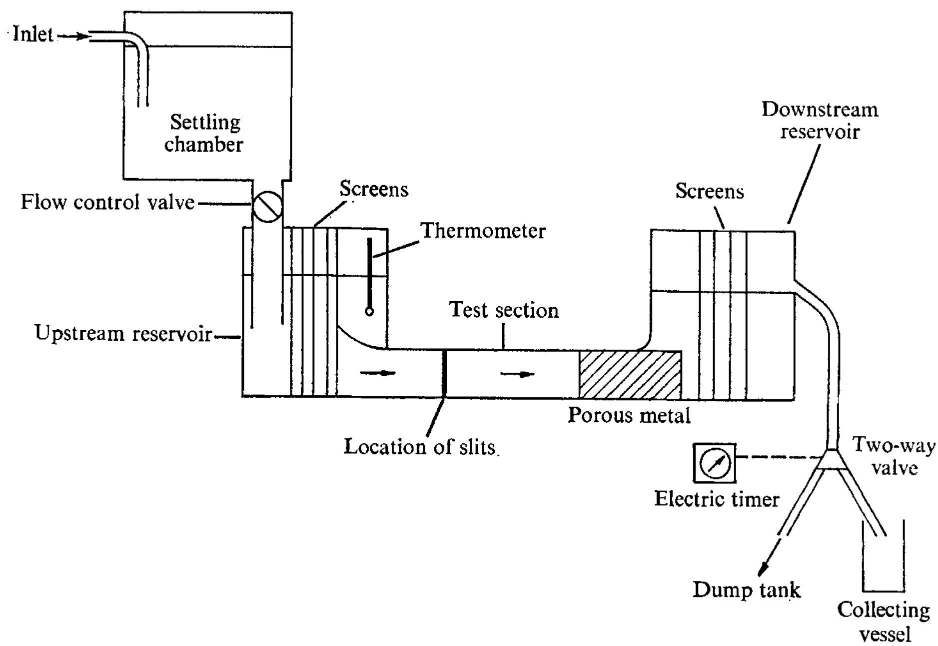

FIGURE 1. Schematic of experimental facility.

continuously from this tank back to the settling chamber by way of a filter. The experiments described herein, however, were exceedingly sensitive to external disturbances (as will be discussed later), which prohibited the use of the pump. The flow into the settling chamber, therefore, was ducted directly from the mains, the flow rate being carefully adjusted so as to maintain the level of the water in the chamber at a constant value.

The temperature of the water entering the test section was monitored throughout each series of experimental runs. The maximum temperature change over a period of about 2 h was 0·5 °F, resulting in a change in viscosity of less than 0·5 %.

Flow Reynolds numbers were determined from the measured mass flow rates through the tunnel. Quantities of water were collected and weighed, while the time to accumulate the samples was measured on a timer which was activated by the operation of the two-way valve.

Two circular orifices, with diameters of 0·320 and 0·625 in., and four slits, with widths of 0·222, 0·300, 0·424, and 0·600 in., were used in this investigation. These were placed in the tunnel test section about 5 in. downstream of the entrance. Both the slits and the orifices had very sharp edges, although preliminary experiments with the slits indicated that the vortex pattern and frequency of shedding were unchanged when the sharp edges were made slightly rounded. Visualization of the flow field downstream of the sharp edges was accomplished by the introduction, via small hypodermic tubes, of neutrally buoyant dye streams into the flow immediately upstream of the edges. The vortex patterns were photographed by means of a camera situated directly above the test section. The vortex shedding frequency was obtained by counting the number of vortices passing a fixed point in a given time for values less than about 200 per min. At higher shedding rates, the frequency was determined using stroboscopic lighting.

Throughout the course of this investigation, care was taken to isolate the equipment from any outside disturbance sources. The facility was set up in a very quiet basement laboratory in which no building vibrations could be detected. Several fine mesh wire screens were installed in both the upstream and downstream reservoirs to dampen any disturbances which may have been generated at the inlet and exit respectively of the two reservoirs. In addition, in order to eliminate any 'U-tube' oscillations of the fluid in the system as a whole, a block of porous nickel foametal, 6 in. long, was installed in the test section about 10 in. downstream of the location of the slits and orifices. While frequency measurements were being taken movements about the laboratory were suspended, and in particular the equipment was not touched in any way.

Experimental results

Typical flow patterns for the two-dimensional and circular jets under a variety of operating conditions are shown in figures 2–5 (plates 1–4). Comparison of these figures with the photographs of vortex streets in jets issuing from nozzles and thick orifices reveals a similarity in the vortex patterns. In particular, figures 2 and 5(*b*) compare quite closely with the pictures from the film by Berger of the vortices in the jet emanating from an axisymmetric nozzle (Wille 1963), although these latter photographs show that the vortex pattern is preserved for much higher Reynolds numbers than in the present experiments. The shadowgraphs of the unexcited jets issuing from circular orifices (Anderson 1956), and the smoke photographs of the unexcited jets from a contoured nozzle (Becker & Massaro 1968) also show that patterns consisting of several vortex rings can be detected at Reynolds numbers which are much higher than the value at which a pattern can be observed from a sharp orifice.

In these experiments, both the two-dimensional and axisymmetric jets exhibit the same general qualitative features with respect to the growth of the vortex patterns. No obvious disturbances can be detected in the dye streams for Reynolds numbers less than about 400. Here the Reynolds number R is based on the average velocity U through the slit or orifice and on a length equal to the diameter d of a circular orifice or twice the width w of a slit. For Reynolds numbers between approximately 400 and 500, small irregularities become apparent in the dye streams. These irregularities occasionally break up and form a small part of a street of vortex rings or pairs which are then swept downstream with the flow. The small bursts of segments of vortex streets occur at random intervals, and only over a very small range of Reynolds numbers. A small increase in the Reynolds number above that at which the effect is first observed results in a flow pattern consisting of an uninterrupted street of vortex rings or pairs. The vortices in general appear to have a uniform spacing for a distance of a few characteristic lengths (i.e. orifice diameters or slit widths) downstream of the orifice or slit. The spacing of the vortices in this region scales like the characteristic length for both geometrical configurations.

The development of the vortex street for the 0·320 in. diameter orifice is shown in figure 2. In cross-section, the orifice has the same shape as the slit shown in figure 3, with the sharp edge forming the front of the orifice. The top photograph of figure 2 shows a typical undisturbed jet, with no visible irregularities. A segment of a vortex street, formed by the sudden bursting process described above, is shown in the second photograph. An unusual aspect of this photograph is the non-symmetry of the pattern at the downstream end of the segment, which could have been caused by a disturbance at the downstream end of the test section propagating upstream. For values of the Reynolds number above 470 (at which this photograph was taken), the pattern consists of a fairly regular stream of vortex rings being generated at the sharp edge and carried downstream. As the rings move downstream, the flow pattern eventually becomes irregular and breaks up, often as a result of one vortex ring catching up with the one ahead and moving inside it. The region of vortex growth gets smaller as the Reynolds number increases, with a consequently smaller number of rings constituting the regular pattern. Flow patterns at various Reynolds numbers up to a value of almost 2000 are shown in the bottom four photographs of figure 2. At higher Reynolds numbers, the number of rings in the regular pattern continues to decrease until at a Reynolds of about 3000 no regular pattern beyond the first vortex ring can be observed. Similar patterns, differing only in length scale, were observed using the 0·625 in. diameter orifice.

The variations of Strouhal number with Reynolds number for the circular orifices are shown in the bottom two parts of figure 6. The Strouhal number is defined as fD/U, where f is the vortex shedding frequency, and D is either the diameter of an orifice or width of a slit. The mean value of the Strouhal number for each orifice turned out to be 0·63, which is shown as a solid line in figure 6. The variation of shedding frequency with Reynolds number for each orifice is presented in figure 7, where the two lines are those obtained using the value of 0·63 for the Strouhal number. These lines appear to be excellent representations of the data points.

Several authors have quoted Strouhal numbers for unexcited circular jets issuing from nozzles and thick orifices. Schade & Michalke (1962), using vortex filament nozzles of different diameters, concluded that the Strouhal number could be expressed by $S = 0{\cdot}0195\sqrt{R}$ for Reynolds numbers between approximately 10^4 and 10^5. Becker & Massaro (1968) used a single nozzle designed to ASME specifications to obtain the relationship $S = 0{\cdot}012\sqrt{R}$ over the Reynolds number range of about 2000 to 20,000. These latter authors also '...tentatively

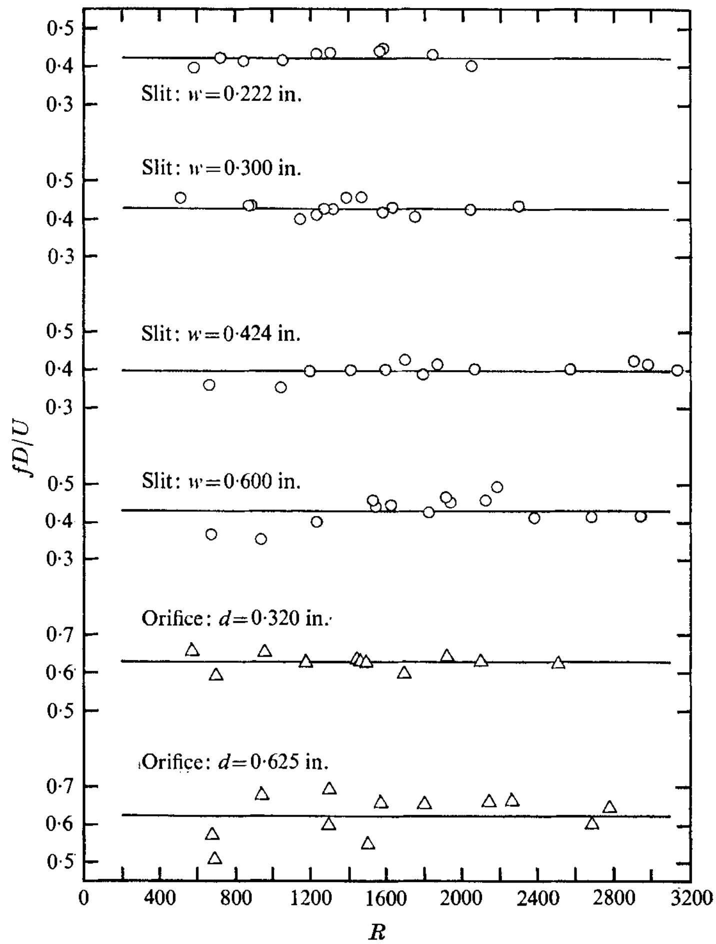

FIGURE 6. Strouhal number against Reynolds number.

conclude that in vortex shedding by an axisymmetrical ducted jet the Strouhal number, in the event of a sufficiently strong positive pressure gradient, has a constant value of about $\frac{1}{2}$'. Johansen (1929) obtained a value of 0·6 for the Strouhal number of a thick circular orifice set in a pipe of twice the orifice diameter for orifice Reynolds numbers between 200 and 1000. Inspection of Anderson's results for jet-tones (1954) reveals Strouhal numbers, based on orifice diameter, in the range 0·5–1·3 for orifice Reynolds numbers between about 3000 and 11,000. The Strouhal numbers show greater consistency when based on the orifice thickness, giving a value of about 0·65. The Strouhal number for Pfeifenton jets (1955*b*) appears to be about 0·66 over the Reynolds number range 4000 to 15,000. It would appear that for nozzle jets at high Reynolds numbers the nozzle design, which determines the boundary-layer thickness at a given Reynolds number, has a significant influence on the value of the Strouhal number. Likewise, the Strouhal number for thick orifices appears to depend on their geometrical configurations. On the other hand, for flows through very sharp orifices at low Reynolds numbers, the wavelength of the vortex street is dictated by the diameter of the orifice. Consequently, the Strouhal number is independent of both Reynolds number and geometric size of the orifice.

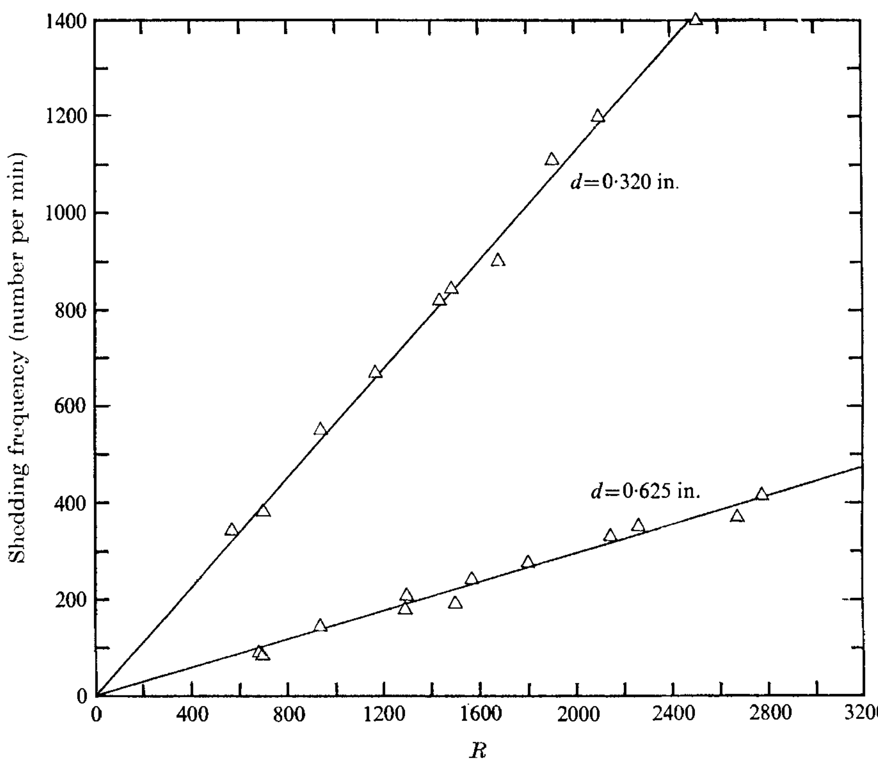

FIGURE 7. Vortex shedding frequency against Reynolds number for circular orifices.

The development patterns of the vortex pairs in the two-dimensional jets show the same qualitative features as those just described for the vortex rings. A sequence of photographs showing the development for the 0·300 in. slit is presented in figure 3, in which it will be observed that the patterns are symmetric even in the small irregularities in the individual vortices. The slight curvature of the jets shown in these and other photographs of two-dimensional jets results from a tendency of the jet to eventually take up a stable position, in which it is attached to one of the side walls of the tunnel test section, unlike the axisymmetric jet, which always issued directly down the centre of the test section. The break-up of the pattern by one vortex pair passing inside another can be seen in figures 3(*c*) and 3(*d*) (plate 2). In 3(*c*), taken at a Reynolds number of 1460, the sixth vortex pair from the sharp edge is about to pass inside the seventh pair, while downstream of those the symmetric pattern has been destroyed. At the higher Reynolds number of 2020, shown in photograph 3(*d*), the fourth vortex pair has passed completely through the fifth vortex pair, and the left and right members of each pair are rotating about one another. Downstream of those, the combined sixth and seventh pairs can be seen breaking up, while even further downstream only one member of the combined original eighth and ninth pairs is still distinguishable.

Figure 4 shows the flow patterns for each of the four slits at Reynolds number in the neighbourhood of 1000. For $w = 0{\cdot}600$ in., the beginning of the regular pattern can be seen about two slit-widths downstream, while for the next smaller slit the regular vortex pattern is clearly developed after about one slit width. The regular pattern of vortex pairs, and the eventual breaking up by the process described earlier, can be seen in the photographs for the two smallest slits, 4(*c*) and 4(*d*).

The variations of Strouhal number with Reynolds number for the slits are shown in the upper four parts of figure 6. The mean value of the Strouhal number for three of the slits ($w = 0{\cdot}222$, 0·300 and 0·600 in.) is 0·43, while the value for the 0·424 in. slit is 0·40. It is thought that this low value may result from a possible small change in the nominal width of the slit, which could have occurred during the original setting-up operation in the tunnel test section. Shedding frequencies are shown as functions of Reynolds number in figure 8, which includes the straight lines obtained from the corresponding mean Strouhal numbers of figure 6.

The constant value of 0·43 obtained here may be compared with results of Michalke & Wille (1964) and Sato (1960). The former authors obtained the relation $S = 0{\cdot}013\sqrt{R}$ for flow from a two-dimensional nozzle with Reynolds numbers between about 10^4 and $2{\cdot}5 \times 10^4$. Sato concluded that the flow from a flat jet gave a constant Strouhal number of 0·23 for Reynolds numbers between about 1500 and 8000 and an increasing value for higher Reynolds numbers.

The vortex patterns in both the two-dimensional and axisymmetric jets are extremely sensitive to external disturbances. This is illustrated in figure 5. Parts (*a*) and (*b*) of this figure show, respectively, the vortex patterns for the smaller and larger orifices when an external exciting frequency of 180 cyc/min is imposed on the flow, this frequency being generated by a vibration in the actual laboratory building in which the experimental equipment was first set up. The corresponding Strouhal numbers are 0·43 and 0·98 respectively, compared with the value of 0·63 for the unexcited flows. The Reynolds number of the flow shown in figure 5(*a*) is 460, compared with the value of 470 for the unexcited flow shown in figure 2(*b*). The latter figure, as described earlier, shows a small segment of a vortex street which has suddenly formed and is carried downstream, whereas the forced flow of figure 5(*a*) shows a continuous stream of vortex rings being generated. A further peculiarity of this pattern is the apparent sudden change in wavelength of the vortex street two or three diameters downstream of the orifice.

The effect of an external disturbance on a two-dimensional jet is shown in figure 5(*c*), in which a building frequency of 880 cyc/min is being imposed on the jet. The Strouhal number for this flow is 0·61, compared with the unforced value of 0·43. An interesting feature of figure 5(*c*) is that at a Reynolds number of 2540 as many as eight distinct vortex pairs can be seen before any sign of break-up is evident, whereas at this same Reynolds number in the unexcited flow no more than two or three distinct pairs can be observed. This persistence of the vortex pattern over greater distances for forced flows has been observed by other authors, who have studied in detail the response of jets issuing from nozzles and thick orifices to both acoustic and mechanical excitation.

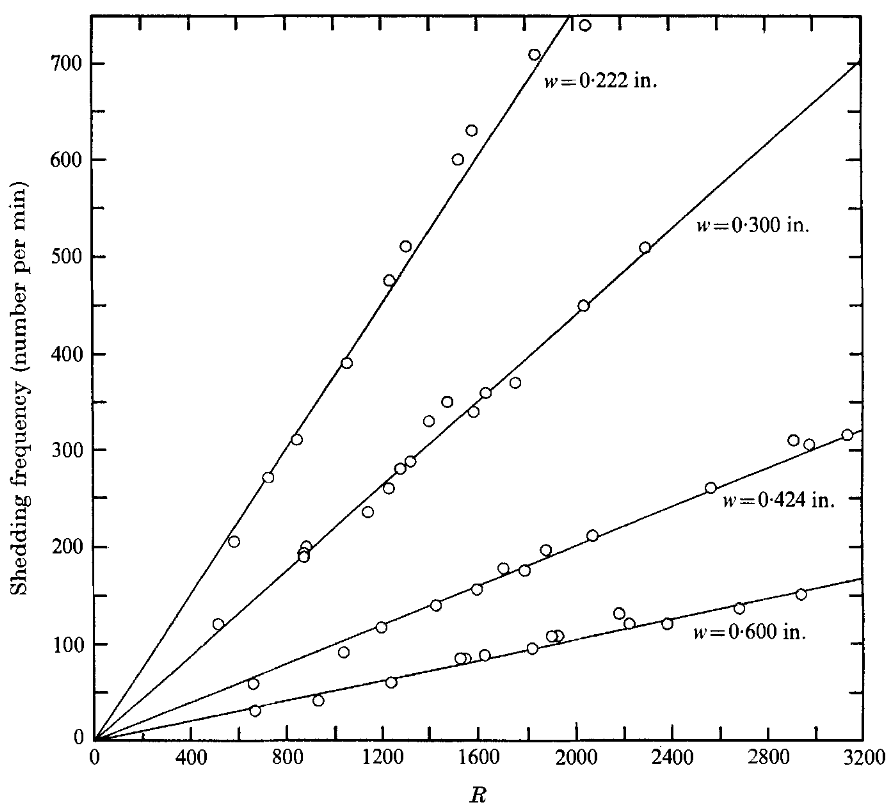

FIGURE 8. Vortex shedding frequency against Reynolds number for slits.

Computer experiments

Two computer experiments were performed using the University of Minnesota CDC 6600 computer in order to determine whether the observed vortex pattern is intrinsic to the vortex growth process and independent of the particular boundary geometry which produces the basic flow. The calculations were set up to simulate the motion of two infinite parallel vortex sheets in an inviscid incompressible fluid when subjected to an initially symmetric disturbance in the first experiment and to an initially non-symmetric disturbance in the second experiment.

The use of numerical techniques to study the motion of continuous vortex sheets represented by discrete arrays of point vortices is not new. The first work of significance in this field was that of Rosenhead (1931), who used a line of point vortices to represent a single vortex sheet in order to compute the effect of a sinusoidal perturbation on the motion of the sheet. He was able to calculate the initial velocities of the points in the sheet and predicted that the motion would result in a rolling up of a vortex. Westwater (1936) used a similar approach to study the rolling up of a vortex sheet behind an airfoil, replacing a single continuous vortex sheet having a particular vorticity distribution by 20 point vortices symmetrically placed about the mid-point of the sheet.

A critical reappraisal of the then available numerical evidence for the rolling up of vortex sheets was made by Birkhoff & Fisher (1959), who questioned whether the behaviour of a continuous vortex sheet is well simulated by discrete vortex arrays. They indicated analytically that discrete vortex arrays may be expected to exhibit a tendency towards randomization of position, and presented numerical calculations for a sinusoidally displaced vortex sheet as evidence of this. Rosenhead's results were further re-examined by Hama & Burke (1960), who expressed some doubt about his computations. By employing smaller time steps and improving the specification of an initial disturbance, they were able to follow the rolling up of a vortex in a single sheet. Abernathy & Kronauer (1962) used a similar model to compute the motion of a pair of vortex sheets after a periodic perturbation of the sheets is introduced. Their work shows that the result of introducing a periodic anti-symmetric disturbance is to produce a periodic asymmetric pattern of vortices. The rolling up of a thick vortex sheet, in which the vorticity distribution in the shear layer is replaced by several lines of point vortices, has been studied by Michalke (1964*a*), who calculated the growth of the vortex pattern after assuming that the pattern possessed a predetermined wavelength. Gerrard (1967) has used the technique of representing separated shear layers by arrays of point vortices to determine the oscillatory variables of the flow past a circular cylinder. An initial asymmetrical vortex distribution was assumed to represent the wake of the cylinder and the point vortices representing the shear layers shed by the cylinder were assumed to appear regularly at predetermined positions in the flow. Gerrard's calculations generated an axisymmetric vortex street with shedding frequencies which compared favourably with available experimental values.

The calculations described in this paper show qualitatively the effects of introducing localized initial disturbances on two infinite initially parallel vortex sheets. Unlike the calculations listed above, no wavelength for the vortex growth process is imposed on the calculations. Each vortex sheet in the present computations is represented by a line of point vortices having a vortex strength π and originally situated one space unit apart along the line. The separation between the lines of points representing the two vortex sheets is three units. An initial disturbance is introduced in the lines, and the motion of the points is calculated by computing the velocity at each point and multiplying by a time interval of $\Delta t = 0{\cdot}1$ to find the displacement to its new position. The motion of the original vortex sheets is then deduced from the motion of these individual vortices.

Before discussing the results of the two calculations, it is appropriate at this point to make some brief comments about the computational procedures. The motions of 100 points in each sheet are computed. The rest of the points are assumed to remain stationary over the duration of the computation. The velocity at a point in the flow is calculated by summing the contributions from all movable point vortices and from 50 stationary point vortices on both ends of each line of movable points. The contributions to the motion from the remaining point vortices, which extend to infinity on both ends of each sheet, are approximated by integrals on these segments of the sheets. The vortex formation process is initiated by applying a suitable disturbance to a few of the movable points in each sheet. The disturbance is such that as few as possible of the movable points are displaced from their initial positions, and yet at the same time the disturbed positions lie on a smooth curve and are of sufficient magnitude as to promote an immediate rolling-up of the first pair of vortices. A final remark concerns the choice of three space units for the distance between the vortex sheets. This minimal spacing is required to allow the formation of a few vortices to be completed within reasonable computation times.

In the first computation, symmetric disturbances are introduced initially into the two vortex sheets, and the subsequent motions of the sheets remain symmetrical. The development of the vortex pattern with time is shown in figure 9, which includes only the motion of the upper sheet. The co-ordinate system in figure 9 is the one in which the fluid velocity is zero far outside the sheets. In the undisturbed situation, the sheets move to the right with a velocity $\frac{1}{2}\pi$ and the fluid between the sheets moves in the same direction with a velocity π. Inspection of figure 9 shows that the initial disturbance rolls up into a vortex. The sheet is weakened upstream of this vortex, and the line moves up and rolls upstream forming a second large vortex. A third vortex begins to form at $t = 10$. The third vortex is much smaller, involving only five points. The fourth vortex, which begins to form at $t = 14$, would again be larger when complete.

An intermediate smoothing operation is necessary in order to permit the calculation to proceed for 17 time units. In the course of the computation, the line of movable points far from the initial disturbance breaks up into a zig-zag pattern with a wavelength of about two units. The existence of this pattern can first be detected at $t = 10$. It is still negligibly small at $t = 12$, but at $t = 14$, the amplitude has reached 1. We think this occurs because the natural instability of a vortex sheet amplifies the noise in the computation. At $t = 10$, the computation is stopped and the line of points upstream of the area of vortex formation is smoothed out. The computation is again stopped at $t = 13$ to smooth out the disturbances which grow as a result of discontinuities in the higher derivatives in the line shape introduced at the end points of the first smoothing.

Some comments can now be made on the vortex formation process shown in

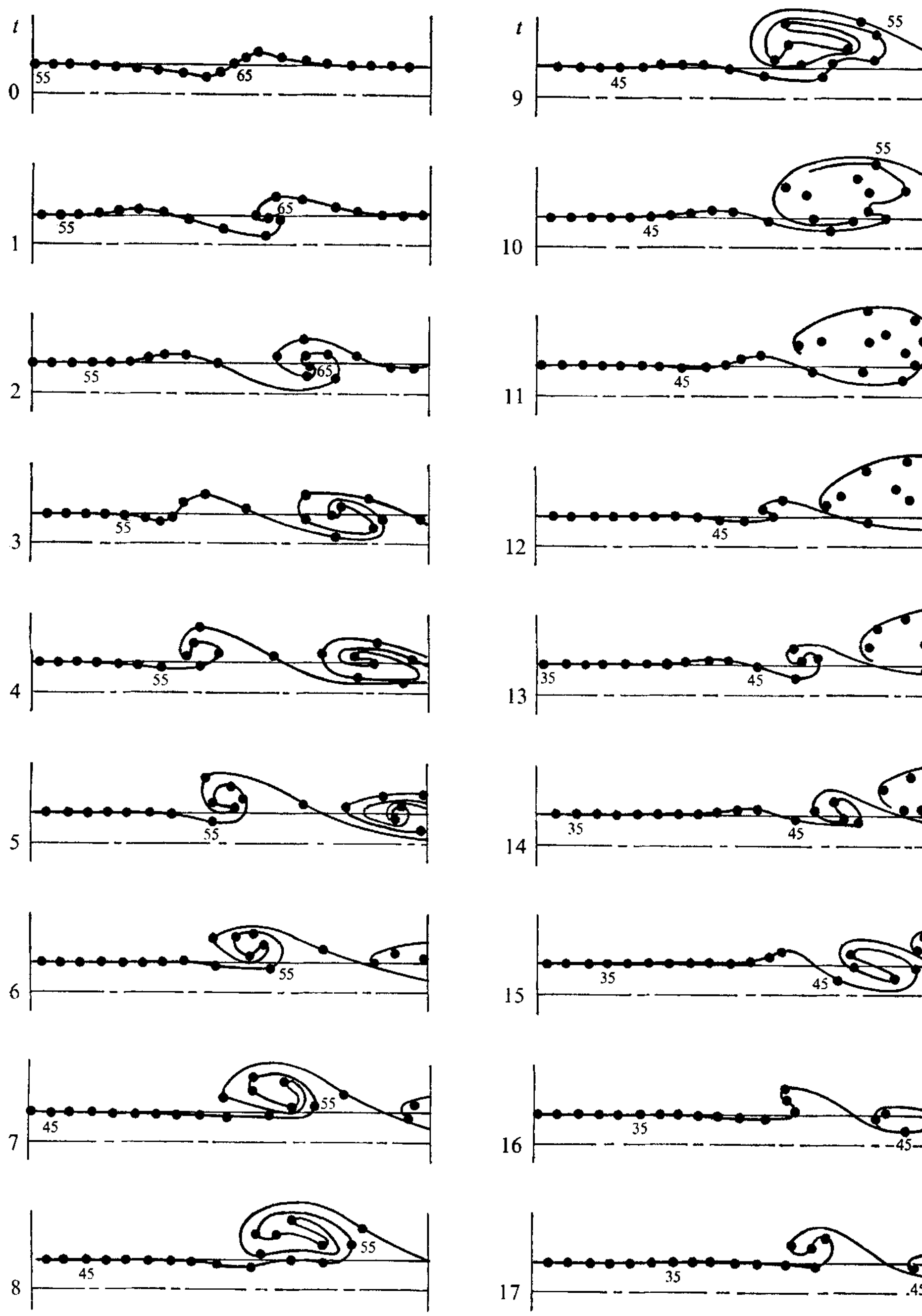

FIGURE 9. Vortex street development from symmetric initial disturbance. The upper sheet only is shown. The numbers refer to initial locations of point vortices.

figure 9. It can be seen that as a vortex pair grows, the rolling up of the vortex stretches the sheet and lowers the vorticity density in the region of sheet near the vortex. If the vortex were growing in a single sheet, where no length scale appears, one could imagine a growth which is similar in time, with the size of the vortex increasing linearly with time without limit. The vortex would have to grow fast enough to continually roll up the stretched part of the sheet nearby. Vortex pairs forming in two parallel sheets, on the other hand, must interact in a way that allows the sheets to spread apart upstream of the stretched region and produce disturbances in the sheets that grow into another vortex pair. This effect does not seem to depend on the vortex pairs interacting to increase the downstream convection speed of the pair. At least the size of this effect is very small for the geometry of the observed vortex pairs.

The size of the vortex pairs formed in this computer experiment does not appear to have much significance. The second and fourth vortices are large because the centres of growth for these vortices are outside the jet and move upstream relative to the sheet. The third vortex forms on the original jet boundary, and finally consists of just five units of the sheet. The process of vortex formation has not yet become regular over the duration of the computation.

The significant feature of this computation is the result that the vortex-formation process propagates fast enough to maintain itself at a fixed position. It is not carried downstream by the jet. The rate of propagation of the disturbance may in fact be underestimated in the computation, because the smoothing of the sheets may remove small upstream effects of the disturbance.

The same form of the initial disturbance as applied to each sheet in the first computer experiment is again applied to each sheet in the second experiment, with the difference that the disturbance in the bottom sheet is displaced by three units along the sheet in order to introduce an asymmetry in the initial flow field. The growth of the vortex pattern is shown in figure 10, up to a time $t = 13$. No smoothing is required in these computations up to this time. It is evident from figure 10 that the region of vortex formation becomes more symmetrical as time proceeds. Qualitatively similar results to those shown in this figure were also obtained for an initial asymmetry of one unit in the locations of the applied disturbances.

In conclusion, we may say that some of the significant features of the flows observed in the laboratory experiments have been reproduced in the computer experiments. The results of the computations show that a region of vortex growth can remain stationary in the flow, and that the vortex growth tends to occur symmetrically. These results suggest that the slits and orifices used in the laboratory experiments serve only as a means of generating the vortex sheets and of providing a reference location for the start of the vortex growth.

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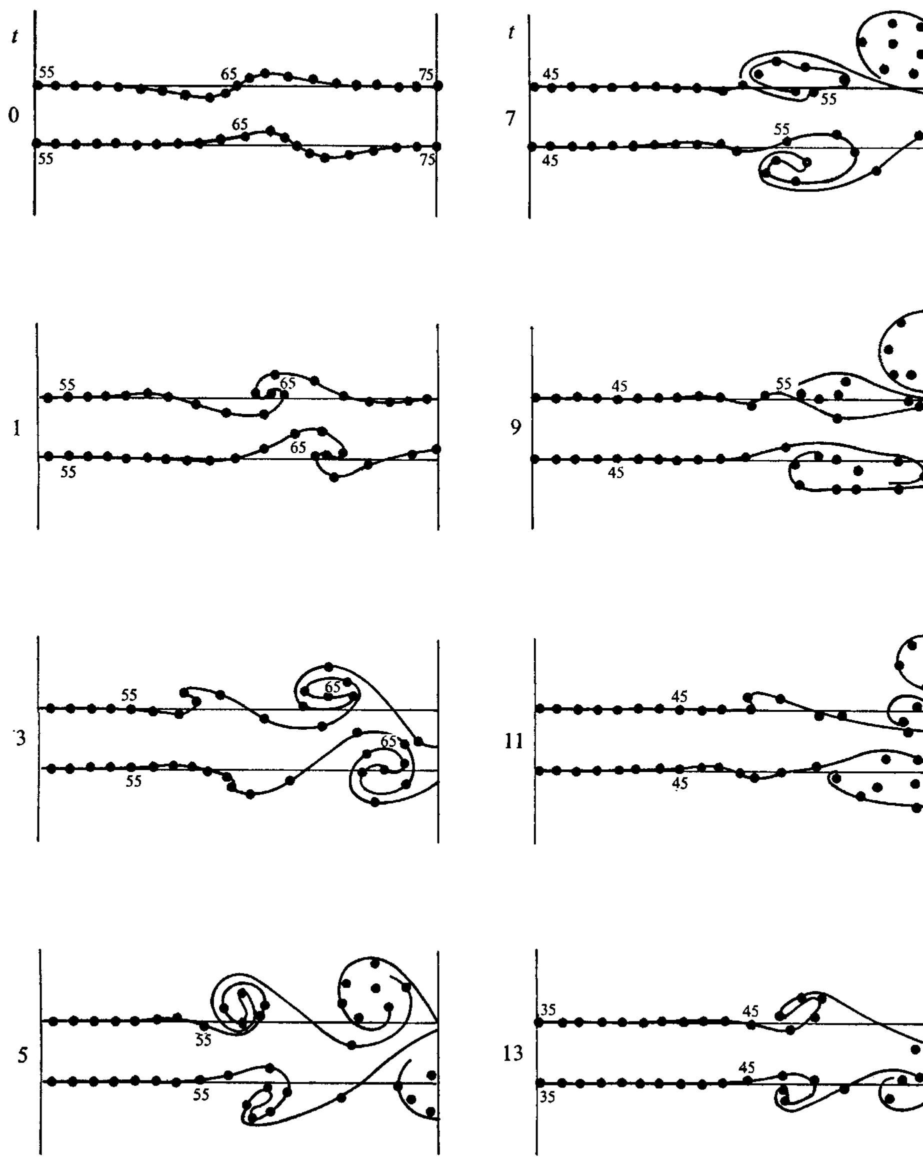

FIGURE 10. Vortex street development for non-symmetric initial disturbance. The numbers refer to the initial locations of the point vortices.

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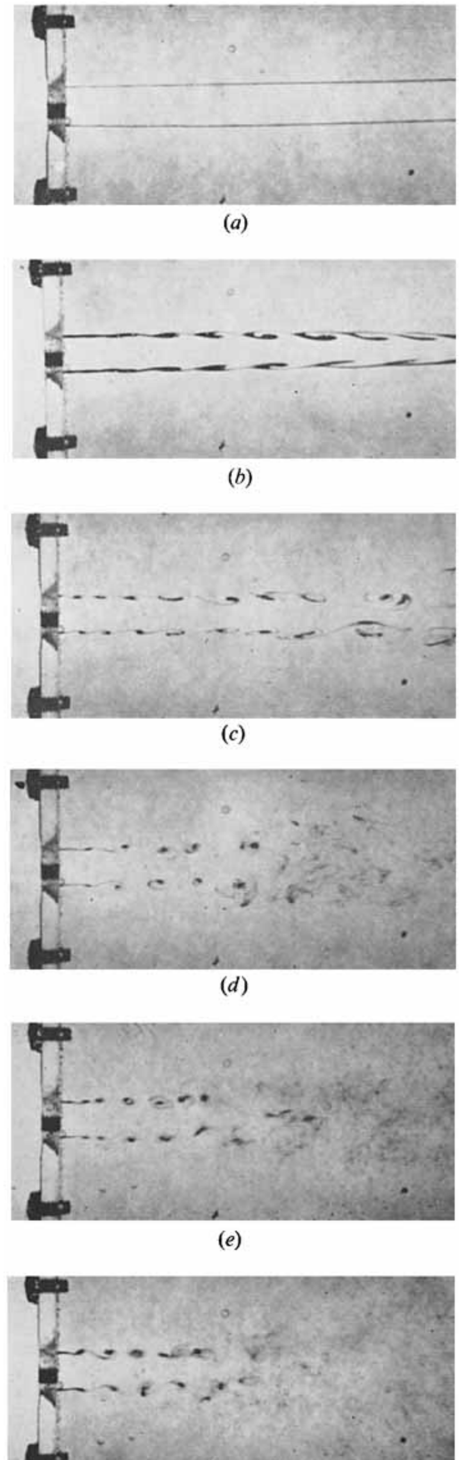
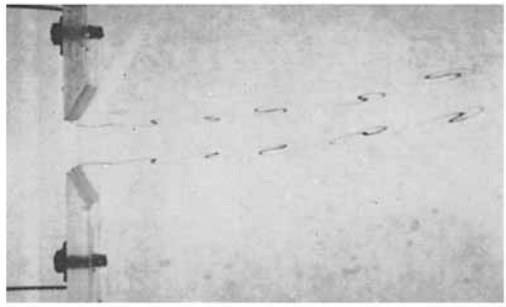
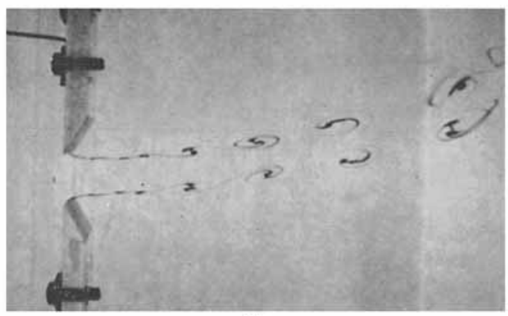


FIGURE 2. Flow patterns in a circular jet: $d = 0{\cdot}320$ in. R: (*a*) 160, (*b*) 470, (*c*) 760, (*d*) 970, (*e*) 1640, (*f*) 1990.



(*a*)



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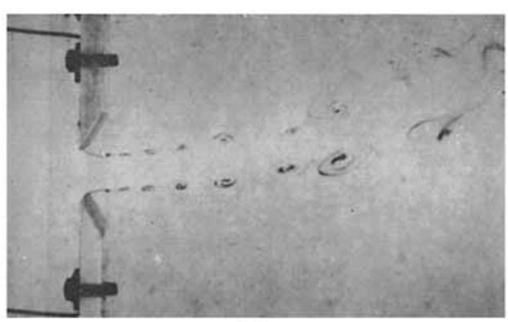
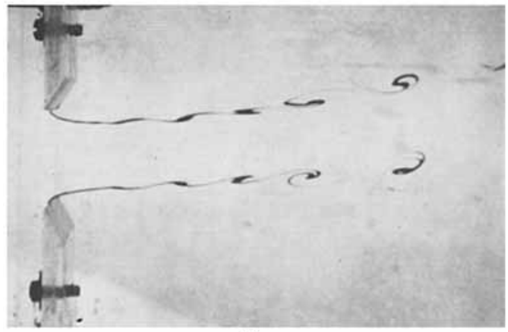


(*c*)

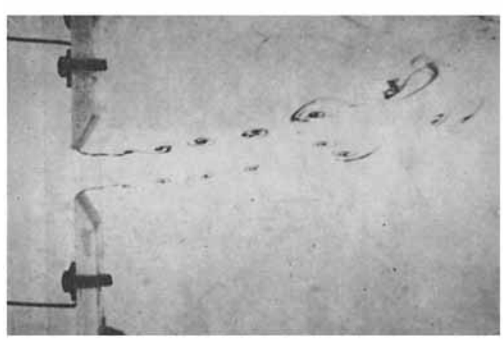
(*d*)

FIGURE 3. Flow patterns in a two-dimensional jet: $w = 0{\cdot}300$ in.
R: (*a*) 510, (*b*) 640, (*c*) 1460, (*d*) 2020.



(*a*)

(*b*)

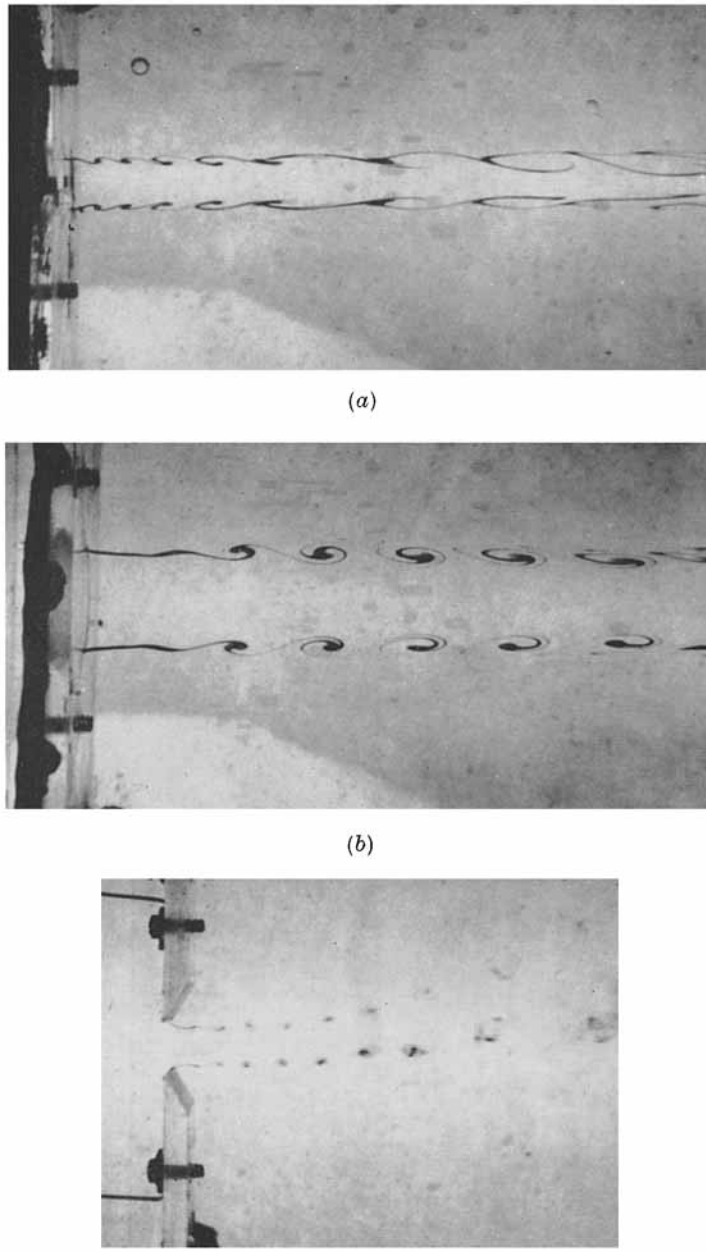


(*c*)

(*d*)

FIGURE 4. Flow patterns in two-dimensional jets at Reynolds numbers near 1000. w (in.), R: (*a*) 0·600, 1140; (*b*) 0·424, 1020; (*c*) 0·300, 1080; (*d*) 0·222, 1020.

BEAVERS AND WILSON



(*a*)

(*b*)

(*c*)

FIGURE 5. Flow patterns in externally excited jets: (*a*) circular jet, $d = 0{\cdot}320$ in., $R = 460$, $f = 180$ per min.; (*b*) circular jet, $d = 0{\cdot}625$ in., $R = 770$, $f = 180$ per min.; (*c*) two-dimensional jet, $w = 0{\cdot}300$ in., $R = 2540$, $f = 880$ per min.